A General Equivalent Circuit for Coupled-Cavity Slow-Wave Structures

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Abstract—A number of structures of the slot-coupled cavity chain type are used in high-power traveling-wave tubes. In these, different relationships between the structure pass bands and the cavity and slot resonances are found to exist. By the construction of equivalent circuits which have patterns of current flow similar to those in the real structures, this behavior is calculated and the importance of the partial nature of the coupling and the relative alignment of the slots is illustrated. It is possible to obtain from the circuits presented a qualitative understanding of the general behavior of structures of this type.

INTRODUCTION

THE SLOW-WAVE transmission-line structures used in high-power traveling-wave tubes and other related devices often take the form of stacks of resonant cavities coupled together by slots in their common walls. Because the boundary conditions are complicated, the analysis of these structures by field theory is very complex and does not give much insight into their behavior. As the author has shown in a previous paper [1], a simple analogous lumped circuit can be used to represent quite accurately the dispersion and impedance properties of this type of structure if it corresponds closely to the geometrical configuration of the structure, and if it takes into account that only part of the current circulating in each cavity may be involved in the coupling. The two circuits dealt with in the earlier paper—"partially coupled" and "staggered"—will be shown to be special cases of a more general circuit, which is useful in understanding the behavior of many types of coupled-cavity structure.

THE GENERAL EQUIVALENT CIRCUIT

A single cavity resonating in its fundamental mode can be represented by a simple resonant circuit C_c, L_c (Fig. 1). The capacitance C_c represents the central part of the cavity which is the region of strong axial electric field, while the inductance L_c represents the outer part of the cavity which is the region of strong magnetic fields and of current flow in the cavity wall. The resonant frequency of the circuit.

$$f_c = 1/2\pi(C_cL_c)^{1/2}$$

is chosen to be equal to the known resonant frequency of the cavity. The characteristic admittance

$$Y_c = (C_c/L_c)^{1/2}$$

is of a more arbitrary nature and its value will depend

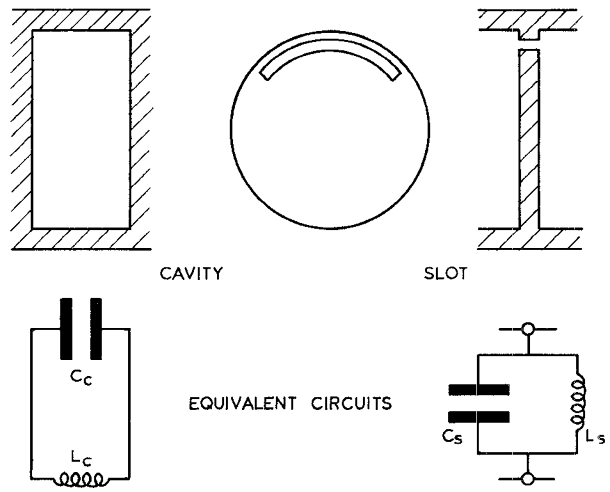

Fig. 1. Cavity and slot equivalent circuits.

on how the voltage on the circuit is assumed to be related to the electric field in the cavity.

The type of structure considered here consists of a stack of such cavities coupled together by slots in their common walls. Central holes may also be cut to allow passage of an electron beam, but the coupling introduced by them would be negligible. The pattern of the slots is assumed to be simple enough for them to be represented by a single terminal pair. The slots themselves form microwave resonant elements and are represented by parallel tuned circuits shunted across the terminal pairs (Fig. 1). For long, thin slots it is appropriate to choose the capacitance C_s and the inductance L_s so that at the resonant frequency

$$f_s = 1/2\pi(C_sL_s)^{1/2}$$

the slots are a half wave length long. The characteristic admittance

$$Y_s = (C_s/L_s)^{1/2}$$

will again be somewhat arbitrary, though it may be related to that of the slots regarded as TEM transmission lines [2]. For the present purpose it is not necessary to regard Y_c and Y_s as accurately known; their ratio will be a significant parameter of the circuit, and its effective value can best be determined experimentally. The slots intercept some of the current circulating in each cavity and make it common to the adjacent cavities. The pattern of the slots cut in each wall will be the same throughout the structure, but the patterns in the two walls of any one cavity may be rotated with respect to one another (Fig. 2). Thus some parts of the slots in one wall will be opposite slots in the other wall, but some

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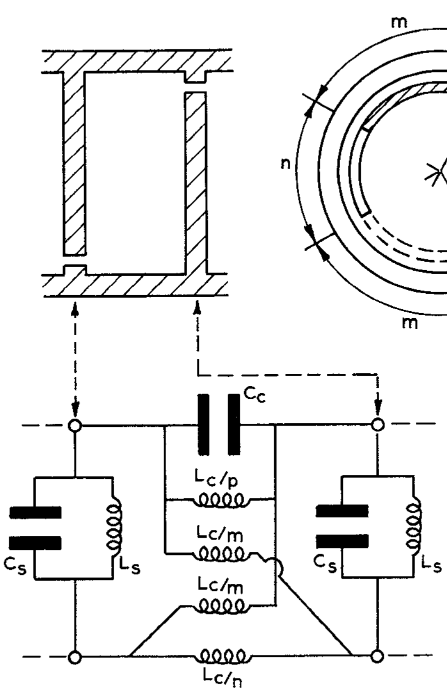

Fig. 2. Complete equivalent circuit.

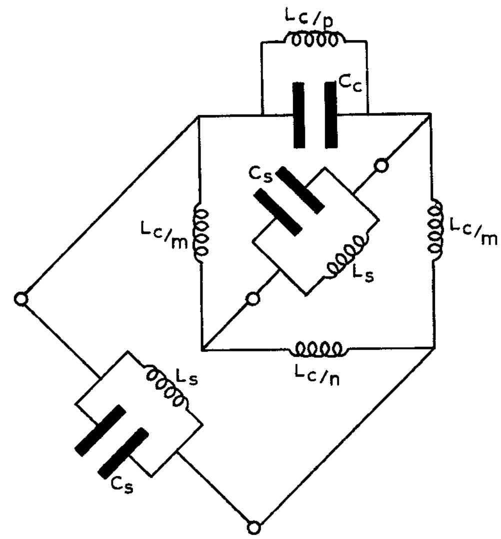

Fig. 3. Complete circuit rearranged.

parts will be opposite unbroken wall. In an uncoupled cavity the current would be distributed among an infinite number of paths. Some of these paths will be broken by the coupling slots, and the assumptions just made imply that each current path will be one of four types, depending upon whether it is 1) unbroken by any coupling slots, 2) broken only by a slot giving coupling to the following cavity of the structure, 3) broken only by a slot giving coupling from the previous cavity of the structure, or 4) broken by two slots giving coupling to the following cavity and from the previous one.

The equivalent circuit of this situation can be constructed by dividing the cavity inductance into four parallel parts with values L_c/p, L_c/m, L_c/m, and L_c/n, where

- p is the fraction of the circulating current not involved in the coupling; type 1)
- m is the fraction involved in coupling one way only; types 2) and 3)
- n is the fraction involved in coupling both ways; type 4).

Since the whole current flow is contained within these paths,

$$p + 2m + n = 1.$$

The circuit through each of these inductances is then broken and connected to the terminals representing the coupling slots. The complete equivalent circuit constructed as just described is shown in Fig. 2. The part of the chain shown represents one cavity and the coupling slots on either side.

The parameters of the structure which are of interest are the transfer constant θ and the characteristic impedance K. When a wave is traveling along the structure, θ is the difference in phase between corresponding quantities measured in adjacent cavities. The impedance K is defined as

$$K = V^2/2P$$

where P is the power carried by the wave and V is the corresponding voltage amplitude appearing across the relevant part of the structure, in this case the center of the cavities. In the equivalent circuit, V will be the voltage appearing across the capacitance C_c. The analysis of the circuit shown in Fig. 2 is straightforward but tedious. The results are

$$\cos\theta = 1 - \frac{[1-(f/f_c)^2][1+am-(f/f_s)^2]}{a[(m+n)^2-n+n(f/f_c)^2]}$$

$$K = -\frac{2(m+n)^2(f/f_c)[1+am-(f/f_s)^2]}{aY_c\sin\theta[(m+n)^2-n+n(f/f_c)^2]^2}$$

where $a = 2(Y_cf_c/Y_sf_s)$.

It will be seen from these formulas that there are three significant frequencies associated with the circuit. There are two frequencies f_1 and f_2 at which $\cos\theta = 1$, $\theta = 0$, and a frequency f_3 at which $\cos\theta$ becomes infinite. These frequencies are given by

$$f_1 = f_c$$

$$f_2 = f_s(1+am)^{1/2}$$

$$f_3 = f_c\left[1 - \frac{(m+n)^2}{n}\right]^{1/2}.$$

The nature of these frequencies can be seen from the equivalent circuit. f_1 is just the cavity resonant frequency, while f_2 is the slot resonant frequency modified by the effect of the L_c/m inductances. If the circuit is rearranged as in Fig. 3, it becomes apparent that f_3 is the frequency at which the bridge is balanced, giving no coupling between the slots. The formulas indicate that there will be two pass bands with their $\theta = 0$ frequencies at f_1 and f_2. The widths of these bands will depend upon the value of a; but if f_3 is real, it will act as a "stop" frequency and thereby limit the width of one of the bands.

Special Cases

The structure dealt with so far is a general and complicated one. The simpler circuits given in the earlier paper [1] are in fact special cases of it, important because they apply to a large number of actual structures.

There are two useful quantities which can be defined as follows. The first is given by

$$k = m + n.$$

k is thus the fraction of the circulating current in one wall of a cavity which is involved in the coupling. If the slots are peripheral, then k will be effectively the fraction of the periphery that they occupy. The second useful quantity is the parameter a_k of the earlier paper which is given by

$$a_k = ak.$$

The first of the special cases is the "in-line partially coupled" case in which the patterns of the slots in the two walls of every cavity have the same orientation so that the circulating current is intercepted either by both sets of slots or by none. The equivalent circuit becomes that of Fig. 4, and the following formulas apply.

In-line:

$$m = 0,\ n = k,\ p = 1 - k.$$

$$f_1 = f_c,\ f_2 = f_s,\ f_3 = f_k = (1 - k)^{1/2} f_c$$

$$\cos\theta = 1 + \frac{[1 - (f/f_0)^2][1 - (f/f_s)^2]}{a_k[1 - k - (f/f_c)^2]}$$

$$K = -\frac{2k(f/f_c)[1 - (f/f_s)^2]}{a_k Y_c \sin\theta[1 - k - (f/f_c)^2]^2}.$$

The second of the special cases occurs when the patterns of the slots in the two walls of each cavity are rotated with respect to one another. Provided that k is less than one half, a position may be found in which no part of one set of slots is opposite any of the other. This gives the case previously referred to as the "staggered" circuit, but which perhaps should be more completely identified as "partially coupled staggered without overlap"! The equivalent circuit for this is shown in Fig. 5, and the following formulas apply.

Staggered $k < 1/2$:

$$m = k, \qquad n = 0, \qquad p = 1 - 2k$$

$$f_1 = f_c, \qquad f_2 = (1 + a_k)^{1/2} f_s = f_s'$$

$$\cos\theta = 1 - \frac{1}{ka_k}[1 - (f/f_c)^2][1 + a_k - (f/f_s)^2]$$

$$K = -\frac{2(f/f_c)[1 + a_k - (f/f_s)^2]}{ka_k Y_c \sin\theta}.$$

A further special case arises in a staggered structure

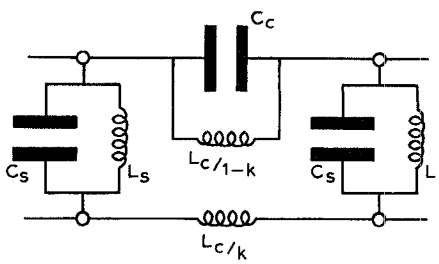

Fig. 4. Partially coupled in-line circuit.

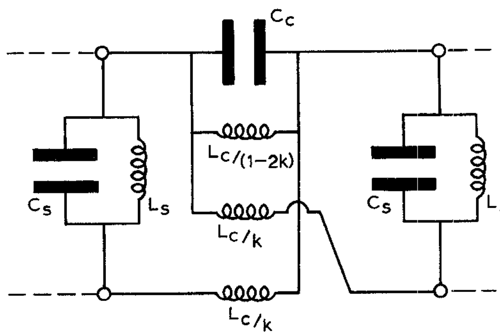

Fig. 5. Partially coupled staggered circuit.

with k greater than one half. Here some parts of the two slot-patterns in each cavity must overlap. The uncoupled inductance L_c/p in Fig. 2 disappears, and the following formulas apply.

Staggered with Overlap $k > 1/2$:

$$m = 1 - k, \qquad n = 2k - 1, \qquad p = 0$$

$$f_1 = f_c,\ f_2 = [1 + a(1 - k)]^{1/2} f_s,\ f_3 = j\frac{(1 - k)}{(2k - 1)^{1/2}} f_c$$

$$\cos\theta = 1 - \frac{[1 - (f/f_c)^2][1 + a(1 - k) - (f/f_s)^2]}{a[(1 - k)^2 + (2k - 1)(f/f_c)^2]}$$

$$K = -\frac{2k^2(f/f_c)[1 + a(1 - k) - (f/f_s)^2]}{aY_s \sin\theta[(1 - k)^2 + (2k - 1)(f/f_c)^2]^2}.$$

Pass-Band Characteristics

The usefulness of the circuits and formulas given in the preceding sections can be determined only by comparing their properties with those of actual structures. In this section the behavior of the pass bands of the circuits will be described, taking a fixed value of a_k ($= 0.5$) and allowing k and the ratio f_s/f_c to vary. (Taking a fixed a_k and varying k is equivalent, for a fixed slot size, to varying the number of slots. In the definition of a_k, Y_s is the admittance of all the slots in parallel so both k and Y_s are proportional to the number of slots.) The frequencies of the band edges can be found from the given formulas. The $\theta = 0$ frequencies are already known as f_1 and f_2, while the $\theta = \pi$ frequencies are found by putting $\cos\theta = -1$. The results are shown in Fig. 6 for the "in-line" case and in Fig. 7 for the "staggered with no overlap" case. In the in-line case, the lower-frequency pass band is influenced by the presence of the "partial" resonance at frequency $f_k = (1 - k)^{1/2} f_c$. In the

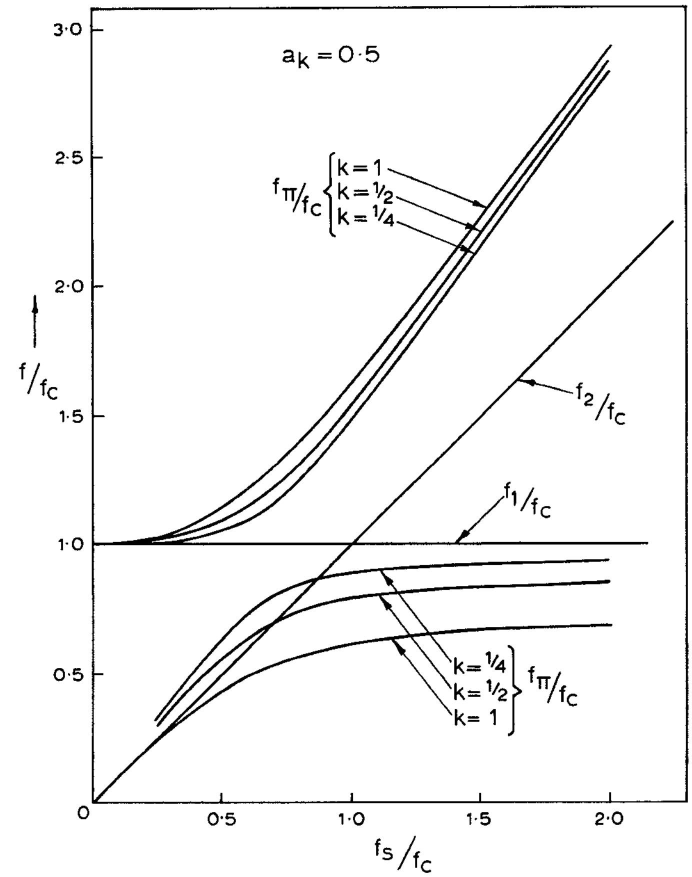

Fig. 6. In-line circuit, band edges.

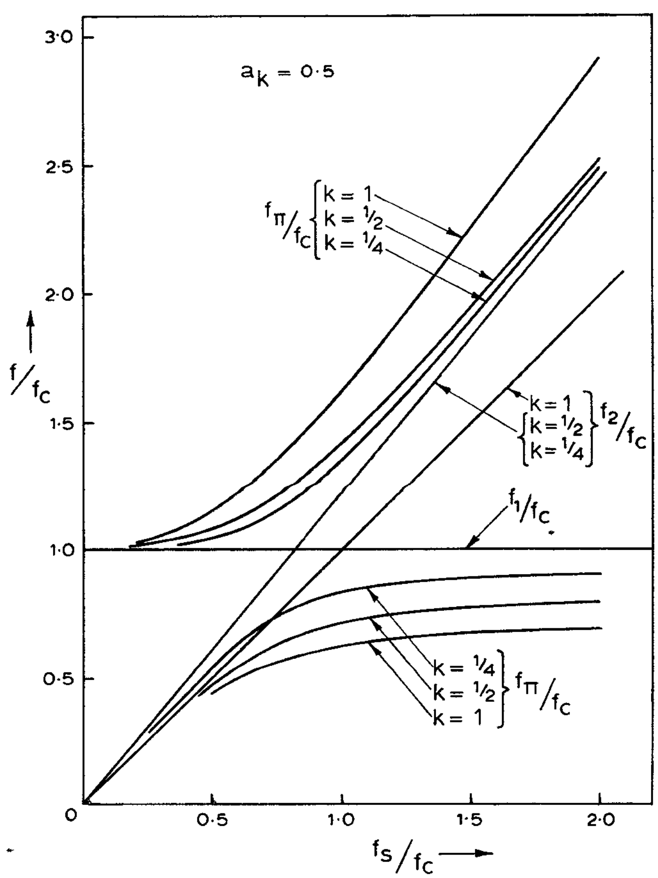

Fig. 7. Staggered circuit, band edges.

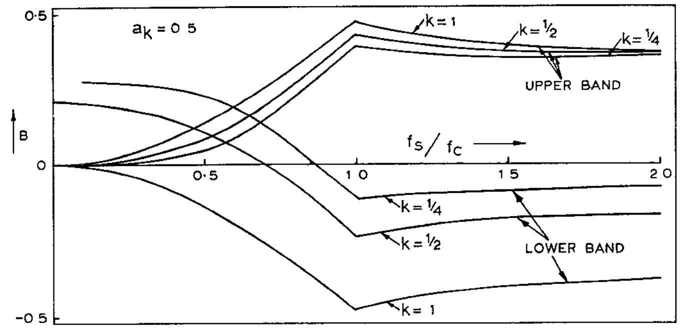

Fig. 8. In-line circuit, bandwidths.

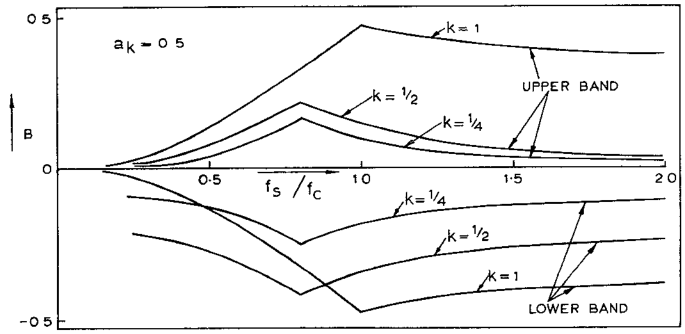

Fig. 9. Staggered circuits, bandwidths.

staggered case, the value of f_2 is greater than f_s with the result that the lower-frequency band is broader than the upper. These same results are displayed again in Figs. 8 and 9 in terms of the bandwidth B. For a band extending from f_0 to f_π this is defined as

$$B = \frac{2(f_\pi - f_0)}{(f_\pi + f_0)}.$$

Thus a negative value for B implies a band with negative group velocity.

Comparison with Real Structures

From the formulas for the pass-band characteristics of the equivalent circuits and the examples of Figs. 6 to 9 some deductions can be made about the behavior to be expected from various types of slot-coupled structure. Some structures have circular cavities and peripheral slots, and these can be dealt with directly by comparison with Fig. 2. The space-harmonic structure of Chodorow and Nalos [3] and the "long-slot" structure of Allen, Kino, and Williams [4] are examples. The "clover-leaf" structure of Chodorow and Craig [5] also uses slot coupling, but the cavities are deformed so that the direction of the coupling is reversed. The equivalent circuit must be modified by reversing the connections between the circuits representing adjacent cavities, and the true dispersion characteristic is obtained by replacing θ by $(\pi-\theta)$; this interchanges the values of f_0 and f_π and so changes the sign of B. The same situation exists in the "centipede" structure [5], although its coupling elements are actually reversed loops.

In a totally coupled structure (in-line with $k=1$) the two pass bands should have their $\theta=0$ frequencies at the cavity and slot resonances; the two bandwidths should be equal in magnitude but opposite in sign, the higher-frequency band having positive group velocity. A totally slot-coupled structure would be difficult to realize in practice since the center portion of the cavity wall would become unsupported. However, this behavior is found in the "centipede" structure [5] when allowance is made for the reversed coupling. The cavity and loop resonant frequencies set the $\theta=\pi$ band edges, and the lower-frequency band has positive group velocity.

A partially coupled in-line structure should behave differently according to whether f_s is greater than f_c, lies between f_c and f_k, or is less than f_k. In the first case, which occurs when the slots are comparatively short, the bands should again have group velocities of opposite sign but the lower-frequency, cavity, band should be narrower than the higher-frequency band. As the admittance ratio parameter a_k is increased, this difference should become greater; since the lower band cannot extend below the partial resonance frequency f_k, its width should become relatively independent of a_k for large values of a_k. This type of behavior is shown, for example, by the space-harmonic structure [3] and the "clover-leaf" structure [5], allowing for the reversed coupling in the latter. In these structures, reducing the height of the cavities or widening the slots, which would increase a_k does not greatly increase the width of the cavity pass band. If the slot resonant frequency is less than the cavity resonant frequency, then the cavity pass band will be the upper one, and this should have positive group velocity and be the broader. The lower-frequency band, extending from its $\theta=0$ frequency f_s toward the partial resonance frequency f_k, should have either positive or negative group velocity as f_s is less than or greater than f_k. This type of behavior is shown in practice by the "long-slot" structure [4], and has been described and explained in terms of field theory by Allen and Kino [2] and by Bevensee [6]. A comparison between the results of the equivalent circuit theory and experiment was given for this structure in the earlier paper [1].

The behavior of staggered structures is characterized by the raising of the $\theta=0$ frequency of the slot pass band from the slot resonant frequency f_s, to an effective value f_s' and by the absence of the partial resonance. This means that the higher-frequency band should generally be narrower than the lower one (Fig. 9) and the width of the latter should be more dependent on the value of a_k than in the in-line case. This indicates that partially coupled staggered structures may be better than their in-line counterparts when it is desirable to use the lower, cavity, pass band since the limitations imposed by the proximity of the slot and partial cavity resonances are absent. This appears to be borne out in practice—as for example, in the "Hughes" structure [7], the Bell M4040 tube [8], and the "modified clover-leaf" structure of Harris [9].

These examples clearly show that many of the properties of the various types of coupled-cavity structure can be understood and predicted from the equivalent circuit.

Acknowledgments

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References

[1] H. J. Curnow, "A new equivalent circuit for coupled cavity structures," *Proc. of 5th Internat'l Congress on Microwave Tubes.* Paris: Dunod, 1965, p. 190.

[2] M. A. Allen and G. S. Kino, "On the theory of strongly coupled cavity chains," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-8, pp. 362–372, May 1960.

[3] M. Chodorow and E. J. Nalos, "The design of high power traveling wave tubes," *Proc. IRE*, vol. 44, pp. 649–659, 1956.

[4] M. A. Allen, G. S. Kino, and C. B. Williams, "A new circuit for wide band high power traveling wave tube," *Nachrichtentech. Fachber.* (*NTF*) (*Germany*), vol. 22, pp. 304–306, 1961. See also E. Belohoubek, "Propagation characteristics of slow-wave structures derived from coupled resonators," *RCA Review*, vol. 19, p. 283, June 1958.

[5] M. Chodorow and R. A. Craig, "Some new circuits for high power traveling wave tubes," *Proc. IRE*, vol. 45, pp. 1106–1118, 1957.

[6] R. M. Bevensee, *Electromagnetic Slow Wave Systems.* New York: Wiley, 1964.

[7] A. W. Scott, "High power TWT's with wide bandwidths," *Electronic Industries*, p. 112, November 1961.

[8] R. J. Collier, G. D. Helms, J. P. Laico, and K. M. Striny, "The ground station high power traveling wave tube," *Bell Sys. Tech. J.*, vol. 42, no. 4, pt. 3, p. 1829, July 1963.

[9] N. W. Harris, "A modified cloverleaf structure for high power travelling wave tubes," *Proc. of 5th Internat'l Congress on Microwave Tubes.* Paris: Dunod, 1965, p. 193.